

Discrete Cat Swarm Optimization for Solving the Quadratic Assignment Problem

^{*1}Mohammed ESSAID RIFFI, ²Abdelhamid BOUZIDI

Laboratory of MATIC, Computer Science Department.

Faculty of Sciences, Chouaib Doukkali University

EL JADIDA, MOROCCO

Email : ¹said@riffi.fr, ²mr.abdelhamid.bouzidi@gmail.com

Abstract. A discrete cat swarm optimization is a metaheuristic based on natural behavior of cats, each cat has two modes that are the seeking mode, and the tracing mode. The seeking mode is when a cat is at rest, that's how a cat spends most of its life time. The tracing mode is when a cat is hunting. This paper proposes a new discrete cat swarm optimization algorithm to solve the quadratic assignment problem, as one of the known combinatorial optimization problems. This problem is attributed to NP-Hard class. In order to test the performance of the algorithm described herein, we will resolve some instances of the quadratic assignment library problem.

Keywords: *Cat Swarm Optimization, CSO, Quadratic assignment problem, QAPLIB, tracing mode, seeking mode.*

* Corresponding Author:
Mohammed ESSAID RIFFI,
Lab. Of MATIC, Dept. of Computer Science, Faculty of Sciences,
Chouaib Doukkali University, EL JADIDA, MOROCCO,
Email: said@riffi.fr Tel: +212-670-471153

1. Introduction

The quadratic assignment problem was introduced in 1957 by C. Koopmans and BECKMAN [1], to solve assignment problems and the location of economic activities [2]. Noticing the large number of area application where this problem appears, such as localization applications [3], Scheduling Problem [4][5], hospital layout [6], computer panel design [7] [8], it ignited interest for many scientific researchers.

For solving this NP-hard combinatorial optimization problem, a set of exact algorithms [9-10], heuristics like Simulated Annealing [11], Tabu Search [12-13], Harmony Search [14], and Metaheuristic based on the natural behavior of swarms are designed to overcome the difficulties presented by conventional methods, like Genetic Algorithms [15-16], Ant Colony Optimization [17], and Particle Swarm Optimization [18].

In this research paper, we will demonstrate how we adapted the cats swarm optimization algorithm for solving the quadratic assignment problem beginning with a presentation of the problem studied. Then we will describe the metaheuristic used, its origin, parameters, and methods. Next, we will propose an adaptation of the cats swarm optimization algorithm in order to solve the quadratic assignment problem and we will present the results for the implementation of this metaheuristic in the standard instances of the library QAPLIB [19,26]. Finally, we will give a conclusion.

2. Formulation of the problem:

The quadratic assignment problem consists of place n objects $O = \{o_1, o_2, \dots, o_n\}$ are intercommunicating in n positions $P = \{p_1, p_2, \dots, p_n\}$. Mathematically it is to realize a bijection α elements of the set O , on those p_j of the set P , with: $(i, j) \in [1, n]$ and $\text{Card}(O) = \text{Card}(P) = n$. A flow

matrix C , where each element c_{ij} indicates the flow between an object o_i and an object o_j . And a distance matrix D where each element d_{kl} gives the distance between a position p_k and a position p_l with $(k, l) \in [1, n]$.

The objective is to minimize the function $F(\varphi)$:

$$F(\varphi) = \sum_{i=1}^n \sum_{j=1}^n (c_{ij} * d_{\varphi(i)\varphi(j)})$$

The solution can be found through many ways. For our case, we will use the instances of QAPLIB library, where the solution is presented by a permutation vector φ , of n elements:

$$\varphi = \{\varphi_1, \varphi_2, \dots, \varphi_n\}$$

The optimal solution will be the optimal permutation vector φ where $F(\varphi)$ has the optimum value. The optimum function values solution are given in QAPLIB web site [26].

3. Cat Swarm Optimization:

The Cat Swarm optimization (CSO) algorithm was realized in 2006 CHU & TSAI[20], and used by other researchers to solve Continuous Optimization for a single objective problem[20-21], and also multi-objective[22-24]. This algorithm was used in 2013 by A.BOUZIDI and M.E.RIFFI [25], to solve a classical combinatorial optimization problem, which is the traveling salesman problem.

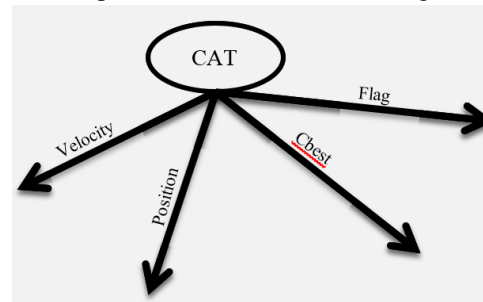


Figure 1 : Characteristics of Cats

Every cat is characterized by its position which represents the solution, velocity, best position, and its flag, that shows the mode of cat, if it's in tracing or searching mode. The mode is changed by the mixer ratio.

4. CSO to solve the Quadratic Assignment Problem:

To solve the Quadratic Assignment Problem (QAP), we need to find the minimum permutation vector into distance matrix D . This vector will be presented as the position of the cat, and the description of every cat in swarm is as following:

- Position** : permutation vector φ
- Cbest** : Best permutation vector found by this cat
- Velocity** : Present the set of permutation to be applied to the position of the cat
- Flag** : used to determinate if the cat is in SM or TM.

After setting the operators, we'll go to the handling part, and the use of characteristics.

As already indicated, each cat has two modes, the searching (SM) and the tracing mode (TM) that are combined by the mixer ratio (MR).

4.1. Searching Mode (SM):

It shows that the cat i is at rest, the parameters used in this mode are:

- SMP**: Seeking memory pool.
- CDC**: seeking range of the selected dimension.
- SRD**: counts of dimension to change
- SPC**: self-position consideration.

The process of seeking mode is described as follows:

Begin

```

Make j copies of catk
if (SPC==true)
    includes catk as a candidate and j=SMP-1
ELSE
    j= SMP
EndIf
For each copy
    Select a number of dimensions based on CDC
    Update their value using SRD percent of their current
value
    Evaluate the fitness of each copy
    For each cat
        
$$P_k = \frac{|FS_k - FS_{min}|}{FS_{max} - FS_{min}}$$

        Randomly select a new position for catk
    EndFor
EndFor

```

END

4.2. Tracing mode TM:

This is the hunting mode, when every cat_i moves quickly to trace his path by updating its velocity, and after that moves according to its velocity position. The description of the process is as follows:

Begin

```

//update_velocity

$$\mathbf{V}'_i = \mathbf{w} * \mathbf{V}_i + \mathbf{r}_1 * \mathbf{c}_1 * (\mathbf{X}_{best} - \mathbf{X}_i) // (1)$$

//update_position

$$\mathbf{X}_i = \mathbf{X}_i + \mathbf{V}_i // (2)$$


```

END

Where, in equation (1):

- \mathbf{X}_{best} : is the best solution / position of the cat who has the best fitness value.
- \mathbf{V}_i : The old speed value (current value).
- \mathbf{V}'_i : the new value of the velocity obtained by the equation.
- \mathbf{c}_1 : is a constant.
- \mathbf{r}_1 : a random value in the range [0, 1]

And in equation (2):

- \mathbf{X}'_k : The new value of the position of the cat_i
- \mathbf{X}_k : The actual position of cat_i
- \mathbf{V}_k : The velocity of cat_i

Some concepts about operation will be introduced.

Definition 1: a movement of cat_k is a swap applied to solution/position of this cat.

Definition 2: Addition between position \mathbf{X} and a velocity \mathbf{V} ($\mathbf{X} + \mathbf{V}$), is applying the swap in \mathbf{V} to position \mathbf{X} , the result is a new position.

Definition 3: Addition between two velocities \mathbf{v} and \mathbf{v}' ($\mathbf{v} + \mathbf{v}'$), is a new velocity containing all the couple of swaps of \mathbf{v} and \mathbf{v}' .

Definition 4: the result of the subtraction between two positions \mathbf{x} and \mathbf{x}' is a velocity \mathbf{v} , it's the opposite of addition:

$$\mathbf{x} + \mathbf{v} = \mathbf{x}' \Leftrightarrow \mathbf{x}' - \mathbf{x} = \mathbf{v}$$

Definition 5: a multiplication is performed between a float value and velocity, the result is a velocity. The different possible cases according to the real k are:

- If $k = 0$: $k * v = 0$
- If $(k > 0 \ \& \ k \leq 1)$: Then $r * v = (i_{k,j,k})[k : 0 \rightarrow (c * |v|)]$
- If $k > 1$: then we separate. Decimal and integer part, $k = n + x$. Where n is the integer part of r , and x corresponds to the decimal parts. We will then return each party to the previous cases.
- If $k < 0$: $k * v = (-k) * -v$. Now $(-k) > 0$, and you will consider one of the previous cases.

4.3.CSO Algorithm:

To combine the two modes of CSO algorithm, the mixture ratio (MR) indicates the rate of mixing the seeking mode and the tracing mode. The general process of Cat Swarm Optimization Algorithm is:

```

BEGIN
  Create N cats
  Analyze the position, velocity, and flag of every cat
  Repeat
    Evaluate the position of each cat and keep the position
    of the cat with the best fitness value.
    If flag of catk is SM
      Apply catk into SM Process
    ELSE
      Apply catk into TM Process
    End If
    Reinitialize the flag of each cat
  Until ( Condition is satisfied )
END

```

5. Results and Discussion:

In this part we will check the validity of the proposed adaptation for the CSO algorithm to resolve QAP problem. For this benchmark **instance** of QAPLIB library is selected for the simulations. The experiments were performed on a PC with processor Intel(R) Core(TM) 2 Duo CPU T5800@ 2.00GHZ and 3.00 MB of RAM.

The table 2 presents the result of applying CSO to some instances of QAPLIB the best Optimum found in the library [26], For Each **instance** runs by a generation of a random number of cats. After reaching to the result **opt obtained** taken in ten iterations, the **time** is the average execution time got by the Sum time of every execution divided by ten, and calculate the average **Err**, the error is got by:

$$\%err = \frac{\text{optimum}_{\text{obtained}} - \text{Optimum}}{\text{Optimum}} \times 100$$

This table 1 show the values of parameters used:

Table 1. Parameter values that has been used

SMP	5
CDC	0.8
MR	0.3
C1	2.05
R1	[0,1]
W	0.729

Table 2. Table of Results

Instance	opt	Opt Obtained	Time (s)	Err
----------	-----	--------------	----------	-----

Bur26a	5426670	5426670	15	0
Bur26b	3817852	3817852	20	0
Bur26c	5426795	5426795	24	0
Bur26d	3821225	3821225	69	0
Bur26e	5386879	5386879	05	0
Bur26f	3782044	3782044	21	0
Bur26g	10117172	10117172	36	0
Bur26h	7098658	7098658	22	0
Chr12a	9552	9552	00	0
Chr12b	9742	9742	01	0
Chr12c	11156	11156	03	0
Chr15a	9896	9896	19	0
Chr15b	7990	7990	01	0
Chr15c	9504	9504	05	0
Chr18a	11098	11098	41	0
Chr18b	1534	1534	01	0
Chr20a	2192	2196	46	0
Chr20b	2298	2298	417	0
Chr20c	14142	14142	05	0
Chr22a	6156	6156	14	0
Chr22b	6194	6194	56	0
Chr25a	3796	3796	140	0
Els19	17212548	17212548	7	0
Esc16a	68	68	00	0
Esc16b	292	292	00	0
Esc16c	160	160	00	0
Esc16d	16	16	00	0
Esc16e	28	28	00	0
Esc16f	0	0	00	0
Esc16g	26	26	00	0
Esc16h	996	996	00	0
Esc16i	14	14	00	0
Esc16j	8	8	00	0
Esc32a	130	130	00	0
Esc32b	168	168	00	0
Esc32c	642	642	00	0
Esc32d	200	200	01	0
Esc32e	2	2	00	0
Esc32g	6	6	00	0
Esc32h	438	438	03	0
Esc64a	116	116	30	0
Esc128	64	64	979	0
Had12	1652	1652	00	0
Had14	2724	2724	00	0
Had16	3720	3720	02	0
Had18	5358	5358	02	0
Had20	6922	6922	01	0
Kra30a	88900	88900	27	0
Kra30b	91420	91420	104	0
Kra32	88700	88700	90	0
Lipa20a	3683	3683	04	0
Lipa20b	27076	27076	00	0
Lipa30a	13178	13178	49	0
Lipa30b	151426	151426	00	0
Lipa40a	31538	31538	733	0

Lipa40b	476581	47 6581	00	0
Lipa50a	62093	62093	2233	0
Lipa50b	1210244	1210244	00	0
Lipa60b	2520135	2520135	00	0
Lipa70b	46032200	46032200	00	0
Lipa80b	7763962	7763962	00	0
Lipa90b	12490441	12490441	00	0
Nug12	578	578	00	0
Nug14	1014	1014	01	0
Nug15	1150	1150	00	0
Nug16a	1610	1610	00	0
Nug16b	1240	1240	01	0
Nug17	1732	1734	16	0
Nug18	1930	1930	05	0
Nug20	2570	2570	09	0
Nug21	2438	2438	05	0
Nug22	3596	3596	06	0
Nug24	3488	3488	08	0
Nug25	3744	3744	07	0
Nug27	5234	5234	22	0
Nug28	5166	5166	132	0
Nug30	6124	6124	961	0
Rou12	235528	235528	01	0
Rou15	354210	354210	01	0
Rou20	725522	725522	988	
Scr12	31410	31410	00	0
Scr15	51140	51140	02	0
Scr20	110030	110030	07	0
Sko42	15812	15812	834	0
Sko49	23386	23418	5271	0.14
Tai12a	224416	224416	01	0
Tai12b	39464925	39464925	01	0
Tai15a	388214	388214	33	0
Tai15b	51765268	51765268	01	0
Tai17a	491812	491812	03	0
Tai20a	703482	708584	96	0
Tai20b	122455319	122455319	19	0
Tai25a	1167256	1167256	1465	0
Tai25b	344355646	344355646	473	0
Tai30b	637117113	637117113	352	0
Tai35b	283315445	283315445	915	0
Tai40b	637250948	637250948	3052	0
Tai64c	1855928	1855928	97	0
Tho30	149936	149936	640	0
Tho40	240516	240516	779	0

6. Conclusion:

Recently, a set of difficult combinatorial problems has led many researchers to solve it by several methods, especially methods based on the population approach. In this research paper, we used the algorithm Cat Swarm Optimization, a recent algorithm, which has shown its performance in the table2 that contains the results obtained by its application to this discrete problem without hybridization. We really hope that it will be applied to other combinatorial problems, not just single-objective ones but also for multi-criterion discreetly problems. Finally, we will be very interested to answer your

questions, not only to improve the meta-heuristics, but also to solve difficult optimization problems that surround us.

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